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We propose a method for generating multipartite entanglement by considering the interaction of a system of N two-level atoms in a cavity of high quality factor with a strong classical driving field. We show that when a judicious choice of the cavity detuning and the applied coherent field detuning is made, vacuum Rabi coupling produces a large number of important multipartite entangled states. We can even produce entangled states involving different cavity modes. Tuning of parameters also permit us to switch from Jaynes-Cummings like interaction to anti-Jaynes-Cummings like interaction.

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Two or more quantum systems are entangled when it is impossible to describe their physical properties by means of a direct product of their respective density operators. Entanglement is a natural consequence of linearity of the associated Hilbert spaces and plays a central role in the measurement problem and the interpretation of the quantum world. In a *Gedankenexperiment* after Schrödinger [1], using the properties of quantum theory, the states of an alive and a dead cat are correlated with two microscopic states of a decaying atomic nucleus. The absence of these unusual states in our everyday experience led to intense interrogation about the validity of quantum theory in macroscopic systems. In the last years, big efforts have been put into the preparation of the so called Schrödinger cat states in the laboratory [2,3], where the extreme cat states have been reduced to mesoscopic quantum states with classical counterparts, the so called coherent states. Decoherence processes, increasing its rate with the size of the system, have been claimed as the inhibiting mechanism for the manifestation of entanglement in macroscopic objects. In this sense, realizing bigger Schrödinger cat states in the laboratory will let us test decoherence by monitoring the respective decay [2,4]. Meanwhile, multiple entangled systems are also considered since they are interesting in connection with quantum information [5–7].

Cavity QED, where atoms interact with a quantized electromagnetic field inside a cavity, have already proved to be a useful tool for testing fundamental quantum properties [8]. A review on entanglement of atoms in the micromaser is given in [9]. In the present work, we demonstrate how a very large number of multipartite entangled states can be generated by adding a strong coherent field to the system. The coherent drive brings in great flexibility in the generation of entangled states as we have the freedom of choosing the detuning and strength of the field. Our method enables us to entangle different atoms, different cavity modes as well as atoms and cavity modes. Furthermore, we demonstrate how the external drive has an effect similar to that of a Ramsey field before and after the cavity [10], if we work in dressed state basis. Remarkably, we also show the realisation of purely anti-Jaynes-Cummings like interaction.

We consider the interaction of a single mode (frequency ω) of a high-Q cavity with a spatially narrow bunch of N two-level atoms (transition frequency ω_o), illuminated by an external classical field (frequency ω_L). The associated Hamiltonian reads

$$H = \hbar\omega_o \sum_{j=1}^{N} {\sigma_j}^\dagger \sigma_j + \hbar\omega a^\dagger a + \hbar\Omega \sum_{j=1}^{N} (e^{i\omega_L t} {\sigma_j}^\dagger + e^{-i\omega_L t}\sigma_j) + \hbar g \sum_{j=1}^{N} ({\sigma_j}^\dagger a + \sigma_j a^\dagger), \tag{1}$$

where $\sigma_j = |g_j\rangle\langle e_j|$ and ${\sigma_j}^\dagger = |e_j\rangle\langle g_j|$ are the spin flip operators down and up, respectively, associated with upper level $|e_j\rangle$ and lower level $|g_j\rangle$ of atom j. a and $a^\dagger$ are the annihilation and creation operators associated with the intracavity photon field. g and Ω, both chosen to be real, are the coupling constants of the interaction of each atom with the cavity mode and with the driving field, respectively. A more realistic model of the cavity mode would include its interaction with a dissipative environment, finite Q, and a thermal bath, finite temperature. In this work, we are interested in the strong coupling regime, $g > \kappa$, where dissipation can be neglected.

We change the description of the system of Eq. (1) to a reference frame rotating with the driving field frequency

$$H^L = \hbar\Delta \sum_{j=1}^{N} {\sigma_j}^\dagger \sigma_j + \hbar\delta a^\dagger a + \hbar\Omega \sum_{j=1}^{N} ({\sigma_j}^\dagger + \sigma_j) + \hbar g \sum_{j=1}^{N} ({\sigma_j}^\dagger a + \sigma_j a^\dagger), \tag{2}$$

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being $\Delta = \omega_o - \omega_L$ and $\delta = \omega - \omega_L$. For the sake of simplicity, in what follows, we set $\Delta = 0$. We define

$$
\begin{aligned}
H^L &= H_o^L + H_{int}^L \\
H_o^L &= \hbar\delta a^\dagger a + \hbar\Omega \sum_{j=1}^{N} ({\sigma_j}^\dagger + \sigma_j) \\
H_{int}^L &= \hbar g \sum_{j=1}^{N} ({\sigma_j}^\dagger a + \sigma_j a^\dagger). \qquad (3)
\end{aligned}
$$

The Hamiltonian H^L in the interaction picture yields

$$
\begin{aligned}
H^I = \frac{\hbar g}{2} \sum_{j=1}^{N} \Big(&|+_j\rangle\langle +_j| - |-_j\rangle\langle -_j| + e^{2i\Omega t}|+_j\rangle\langle -_j| \\
&- e^{-2i\Omega t}|-_j\rangle\langle +_j| \Big) a e^{-i\delta t} + \text{H. c.}, \qquad (4)
\end{aligned}
$$

where the dressed states $|\pm_j\rangle = (|g_j\rangle \pm |e_j\rangle)/\sqrt{2}$ are eigenstates of $(\sigma_x)_j = \sigma_j^\dagger + \sigma_j$ with eigenvalues ± 1, respectively. We consider now the strong driving regime, $\Omega \gg \{g, \delta\}$, so we can realize a rotating wave approximation and eliminate from Eq. (4) the terms that oscillates with high frequencies, resulting in

$$
\begin{aligned}
H_{eff} &= \frac{\hbar g}{2} \sum_{j=1}^{N} \Big(|+_j\rangle\langle +_j| - |-_j\rangle\langle -_j| \Big) (a e^{-i\delta t} + a^\dagger e^{i\delta t}) \\
&= \frac{\hbar g}{2} \sum_{j=1}^{N} (\sigma_j^\dagger + \sigma_j)(a e^{-i\delta t} + a^\dagger e^{i\delta t}). \qquad (5)
\end{aligned}
$$

A remarkable feature of Eq. (5), more evident if we choose $\delta = 0$ and $N = 1$, is the simultaneous realization of a Jaynes-Cummings (JC) and an anti-Jaynes-Cummings (AJC) interaction, appearing naturally in trapped ions [11] but not in the context of cavity QED.

We show some examples of the possible applications of the interaction described in Eq. (5). If at $t = 0$, $N = 1$, the 1-atom-field state is $|g\rangle|0\rangle = (|+\rangle + |-\rangle)|0\rangle/\sqrt{2}$, the evolved state after a time t will be

$$
\frac{1}{\sqrt{2}} (|+\rangle|\alpha\rangle + |-\rangle| - \alpha\rangle), \qquad (6)
$$

with $\alpha = g(e^{i\delta t} - 1)/2\delta$. The microscopic-mesoscopic entangled state of Eq. (6) is usually called Schrödinger cat state. Clearly, for the simpler case $\delta = 0$, we have $\alpha = -igt/2$, which shows a fast resonant generation of Schrödinger cat states when compared with dispersive methods [2]. Note that rewriting Eq. (6) in the bare basis and in the Schrödinger picture,

$$
\begin{aligned}
\frac{1}{2} \Big[&|g\rangle (e^{-i\Omega t}|\alpha e^{-i\omega t}\rangle + e^{i\Omega t}| - \alpha e^{-i\omega t}\rangle) \\
&+ e^{-i\omega_o t}|e\rangle (e^{-i\Omega t}|\alpha e^{-i\omega t}\rangle - e^{i\Omega t}| - \alpha e^{-i\omega t}\rangle) \Big], \qquad (7)
\end{aligned}
$$

show that a measurement of the atomic state will produce the so called even or odd coherent states in the cavity field, depending if $|g\rangle$ or $|e\rangle$ was found, respectively. Throughout this work, we specify when the final states are written in the Schrödinger picture, as in Eq. (7), for illustrating the unusual way in which phases appear after the convenient realized transformations. If at $t = 0$, $N = 2$, the 2-atom-field state is $|g_1 g_2\rangle \otimes |0\rangle$, the evolved state after a time t will be

$$
\frac{1}{2} \Big[|\phi_1\rangle|2\alpha\rangle + |\phi_2\rangle|-2\alpha\rangle + (|\phi_3\rangle + |\phi_4\rangle)|0\rangle \Big], \qquad (8)
$$

where $|\phi_i\rangle$ are the eigenstates of the atomic operator $\sum_{j=1}^{2} (\sigma_j^\dagger + \sigma_j)$ with eigenvalues $\gamma_{1,2} = \pm 2$ and $\gamma_{3,4} = 0$. The state of Eq. (8) is a bigger and more elaborated microscopic-mesoscopic 2-atom-field entangled state. Measuring the two atoms in the state $|g_1 g_2\rangle$ will produce, in the Schrödinger picture, the field state

$$
\mathcal{N} \Big(e^{-2i\Omega t}|2\alpha e^{-i\omega t}\rangle + e^{2i\Omega t}| - 2\alpha e^{-i\omega t}\rangle + 2|0\rangle \Big), \qquad (9)
$$

which is a triple mesoscopic field superposition state with not only an "alive" or "dead" cat, but also and mainly "absent". The effective interaction of Eq. (5) lets us create more sophisticated and bigger microscopic-mesoscopic N-atom-field entangled states and field superposition states, that we will not describe here.

When $\delta = \pm 2\Omega$ and $|\delta| \gg g$, Eq. (4) has other interesting limits, respectively,

$$
\begin{aligned}
H_{JC}^{(+)} &= \frac{\hbar g}{2} \sum_{j=1}^{N} \Big(|+_j\rangle\langle -_j|\, a + |-_j\rangle\langle +_j|\, a^\dagger \Big) \\
H_{AJC}^{(-)} &= \frac{\hbar g}{2} \sum_{j=1}^{N} \Big(|-_j\rangle\langle +_j|\, a + |+_j\rangle\langle -_j|\, a^\dagger \Big), \qquad (10)
\end{aligned}
$$

which represent, in the $|\pm_j\rangle$ atomic dressed basis, an effective implementation of a Jaynes-Cummings or an anti-Jaynes-Cummings interaction. As is known, the anti-JC interaction does not appear naturally in the context of cavity QED, where the JC model discard high frequency oscillating terms, the so called counter rotating terms, with no relevant contribution to the system evolution. Another remarkable feature of the interactions described in Eqs. (10) is that they produce, in a controlled manner, the absorption and emission of an intracavity photon while preserving the energy mean value of the atomic state. Clearly, the apparent imbalance in the energy stems from the external driving field, which, surprisingly, is intense enough for being considered as classical. Furthermore, imagine the situation where the atom is initially in the ground state $|g\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle)$ and performs a JC interaction in the atomic dressed basis $|\pm\rangle$, following any of the Hamiltonians described in Eqs. (10). Then, by measuring the atom at the end of the interaction in the atomic bare basis $\{|g\rangle, |e\rangle\}$, our procedure is equivalent to a conventional JC evolution with two Ramsey zones, one before and one after the atom-cavity interaction. This result makes Ramsey zones unnecessary and may open new possibilities for phase-sensitive measurements [12] in closed cavities, as we discuss later.

We concentrate now on the possibilities of the studied scheme when considering the interaction of a bunch of N atoms interacting with two quasiresonant normal modes present in the cavity, always assisted by a strong external driving field. The associated Hamiltonian, in the same spirit of Eq. (2), with $\Delta = 0$ and Ω real, reads

$$H^{ab} = \hbar\delta_a a^\dagger a + \hbar\delta_b b^\dagger b + \hbar\Omega \sum_{j=1}^{N} ({\sigma_j}^\dagger + \sigma_j) + \hbar g_a \sum_{j=1}^{N} ({\sigma_j}^\dagger a + \sigma_j a^\dagger) + \hbar g_b \sum_{j=1}^{N} ({\sigma_j}^\dagger b + \sigma_j b^\dagger), \quad (11)$$

where $\{a^\dagger, b^\dagger\}$ and $\{a, b\}$ are the creation and annihilation operators associated with the two cavity modes, while $\delta_a = w_a - w_L$ and $\delta_b = w_b - w_L$. We define now

$$H^{ab} = H^{ab}_o + H^{ab}_{int} \quad (12)$$

with

$$H^{ab}_o = \hbar\delta_a a^\dagger a + \hbar\delta_b b^\dagger b + \hbar\Omega \sum_{j=1}^{N} ({\sigma_j}^\dagger + \sigma_j)$$

$$H^{ab}_{int} = \hbar g_a \sum_{j=1}^{N} ({\sigma_j}^\dagger a + \sigma_j a^\dagger) + \hbar g_b \sum_{j=1}^{N} ({\sigma_j}^\dagger b + \sigma_j b^\dagger). \quad (13)$$

In the interaction picture, defined by the formal separation in Eq. (13), the Hamiltonian changes to

$$\tilde{H}^{ab} = \frac{\hbar}{2} \sum_{j=1}^{N} \Big(|+_j\rangle\langle +_j| - |-_j\rangle\langle -_j| + e^{2i\Omega t}|+_j\rangle\langle -_j| - e^{-2i\Omega t}|-_j\rangle\langle +_j| \Big) (g_a a e^{-i\delta_a t} + g_b b e^{-i\delta_b t}) + \mathrm{H.c.} \quad (14)$$

In the strong driving limit, $\Omega \gg \{g, \delta_a, \delta_b\}$, we can realize a rotating wave approximation, yielding

$$H^{ab}_{eff} = \frac{\hbar}{2} \sum_{j=1}^{N} (\sigma_x)_j [g_a(a e^{-i\delta_a t} + a^\dagger e^{i\delta_a t}) + g_b(b e^{-i\delta_b t} + b^\dagger e^{i\delta_b t})]. \quad (15)$$

The Hamiltonian of Eq. (15) will produce states with similar features as those produced by the Hamiltonian of Eq. (5), only that now the displacement will act simultaneously on each one of the two cavity modes. If at $t = 0$, with $N = 1$ and $\delta_a = \delta_b = 0$ (for simplicity), the initial atom-field state is $|g\rangle|0\rangle|0\rangle = (|+\rangle + |-\rangle)|0\rangle|0\rangle/\sqrt{2}$, the evolved state at time t will be

$$\frac{1}{\sqrt{2}} \Big(|+\rangle|\alpha\rangle|\beta\rangle + |-\rangle| - \alpha\rangle| - \beta\rangle \Big), \quad (16)$$

with $\alpha = g_a t/2$ and $\beta = g_b t/2$. If $g_a = g_b$, we would have $\alpha = \beta$. Eq. (16) describes an elaborated tripartite entangled state involving one microscopic and two mesoscopic systems. If we measure the atomic state in the bare basis $\{|g\rangle, |e\rangle\}$, we will find the field, in the Schrödinger picture, in the so called entangled coherent states [13]

$$\mathcal{N}^{\pm}_{ab}\big(e^{-i\Omega t}|\alpha e^{-i\omega t}\rangle|\beta e^{-i\omega t}\rangle \pm e^{i\Omega t}| - \alpha e^{-i\omega t}\rangle| - \beta e^{-i\omega t}\rangle\big), \quad (17)$$

respectively. These states have been recently proposed as an important tool in theory and experiments related to the field of quantum information [14]. To our knowledge, it is the first theoretical proposal for producing entangled coherent states in cavity QED, where the quantized field modes are stationary, long-lived and controllable.

As before, we take Eq. (14) to the other limit, where $\delta_a = \delta_b = \pm 2\Omega$. The resulting Hamiltonians, setting $g_a = g_b$ for simplicity, are

$$H^{(+)}_{ab} = \frac{\hbar g}{2} \sum_{j=1}^{N} \Big[|+_j\rangle\langle -_j| \, (a + b) + |-_j\rangle\langle +_j| \, (a^\dagger + b^\dagger) \Big]$$

$$H^{(-)}_{ab} = \frac{\hbar g}{2} \sum_{j=1}^{N} \Big[|-_j\rangle\langle +_j| \, (a + b) + |+_j\rangle\langle -_j| \, (a^\dagger + b^\dagger) \Big]. \quad (18)$$

As we will see, these Hamiltonians are able to produce other kind of interesting nonclassical states. For example, if at $t = 0$, and with $N = 1$, the atom-field state is $|+\rangle|0\rangle|0\rangle$, then, after a time interval t of the $H^{(+)}_{ab}$, the atom-field state will be

$$\cos(\sqrt{2}gt)|+\rangle|0\rangle|0\rangle - i\sin(\sqrt{2}gt)|-\rangle \frac{(|0\rangle|1\rangle + |1\rangle|0\rangle)}{\sqrt{2}}. \quad (19)$$

It is easy to show that if we set the interaction time to be $\tau = \sqrt{2}\pi/4g$, the final state will only contain the entangled two mode field state

$$\frac{1}{\sqrt{2}}(|0\rangle|1\rangle + |1\rangle|0\rangle). \quad (20)$$

This state is a maximally entangled state between the two cavity modes in the $\{|0\rangle, |1\rangle\}$ subspace. This entangled state has been produced recently in cavity QED with a sequence of differently tuned interactions of a single atom with two cavity modes [15]. Eq. (14) may also accept the limit $\delta_a = -\delta_b = \pm 2\Omega$, combining JC evolution in one mode and anti-JC evolution in the other one, producing states that are beyond our present scope. Note that a number of other interesting situations can arise by choosing a nonzero detuning between the external field and the atomic transition frequency ($\Delta \neq 0$).

So far, all previous results are suitable for their implementation in the microwave and optical regime in cavity

QED experiments, with atoms flying through the cavities or conveniently trapped inside them. The microwave regime in the strong coupling limit, involving high-Q cavities and long-living Rydberg atomic levels, is more adequate when considering the production and measurement of the proposed nonclassical states. The optical regime, even if enjoying more restricted strong coupling conditions, may profit from the newly designed interactions in an experimental setup that incorporates comfortably external driving [16–18]. Open cavities, in both regimes, offer the best possibilities for an experimental realization, as long as the direct illumination of the atoms with an external classical field is concerned. Nevertheless, closed microwave cavities with an external field directly coupled to the cavity mode, instead of directly coupled to the atoms, could be also used, profiting from its formal equivalence with the present scheme [19]. This alternative setup would turn the interactions described in Eqs. (10) into a practical realization of a closed high-Q cavity plus two Ramsey zones (without them), combining the advantages of these two powerful tools. In the case of an open cavity, it can be envisaged to add another microwave cavity, always transversal to the crossing atoms, that would maintain them continuously driven, satisfying the requirements of the proposed scheme. Spatially narrow bunches of atoms flying through microwave cavities, or just a few of them, have already been implemented in the laboratory for different purposes [20,21].

The question about the possible implementation of these ideas in the context of trapped ions is relevant. A careful study of Eq. (1) shows that this interaction could be implemented in a simple way, by means of a simultaneous carrier and red sideband excitation, currently achieved in the laboratory. The condition of "strong driving" would be easily satisfied as long as the red sideband excitation is implemented in a strict Lamb-Dicke regime.

We have shown that an additional driving field coupled to the atoms (or to the cavity mode), in the usual cavity QED experiments, can be an important tool for generating multipartite nonclassical states. In particular, we gave examples of how to produce different scalable families of microscopic-mesoscopic N-atom-field entangled states, from which the usually called Schrödinger cat states are a particular case. We showed that under strong detuning conditions it is possible to engineer, in a dressed basis, Jaynes-Cummings or anti-Jaynes-Cummings interactions, being the latter atypical in the context of cavity QED. Phase sensitive measurements of the intracavity field appear naturally in this context. Finally, we discussed an extension of this scheme to the case of two quasiresonant cavity modes, opening the possibility for creating two-mode entangled coherent states and maximally entangled states. Possible applications to quantum information devices are under current research.

Controlled generation of different classes of multipartite nonclassical states are intensely pursued in different physical systems, with the hope of better understanding the fundaments of quantum nature and its properties. By scaling up these special states to bigger ones and to multipartite systems, it is expected to unveil the secrets of decoherence processes and the emergence of the classical world from a quantum picture.

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